# Integrated Modeling for Location Analysis

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# Integrated Modeling for Location Analysis

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Full text is available at http://dx.doi.org/10.1561/0200000037

# Foundations and Trends® in Technology, Information and Operations Management

*Published, sold and distributed by:*
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

*Outside North America:*
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

H.Y. Mak and Z.J.M. Shen. *Integrated Modeling for Location Analysis*. Foundations and Trends® in Technology, Information and Operations Management, vol. 9, no. 1-2, pp. 1–152, 2016.

ISBN: 978-1-68083-191-7
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# Foundations and Trends® in Technology, Information and Operations Management

Volume 9, Issue 1-2, 2016

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Full text is available at http://dx.doi.org/10.1561/0200000037

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Foundations and Trends® in Technology, Information and Operations Management, 2016, Volume 9, 4 issues. ISSN paper version 1571-9545. ISSN online version 1571-9553. Also available as a combined paper and online subscription.

Full text is available at http://dx.doi.org/10.1561/0200000037

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# Integrated Modeling for Location Analysis

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## ABSTRACT

Delivery of products and services relies on well-managed operations. In designing large-scaled supply chain and service systems, locations of key facilities are a critical decision, as these facilities form the backbone of operations of these systems. For example, a key to effective supply chain management is the deployment of a structurally well-designed facility network, consisting of plants, warehouses, retail stores, etc. The aim of the study of facility location is to develop analytical methodologies to inform the planning decisions for evaluating and selecting siting plans for these facilities that ensure both convenient provision of (or access to) products and services by customers and users, as well as efficient operations (i.e., low operating costs).

Facility location and network design has long been an integral topic of study in operations management. In this literature, one may observe that earlier works mainly focused on a strategic view of accessibility and operational costs, using performance metrics based on strategic distances between the chosen facilities and customers or suppliers. This traditional approach often neglects the impacts of future tactical and operational activities to be conducted in the network, and optimizes objectives that do not fully reflect the long-term performance of the facility network. In attempt to rectify this shortcoming, researchers have proposed an integrated modeling approach that enhances the classical models

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Ho-Yin Mak and Zuo-Jun Max Shen (2016), “Integrated Modeling for Location Analysis”, Foundations and Trends® in Technology, Information and Operations Management: Vol. 9, No. 1-2, pp 1–152. DOI: 10.1561/0200000037.

by jointly considering strategic, tactical and operational activities in facility systems. By integrating tactical and operational characteristics of facility networks into strategic design decisions, the integrated approach offers a more balanced perspective on the strategic trade-offs in network design.

As shown in a series of recent research, this integrated modeling approach can potentially deliver new insights into facility location problems in a variety of contexts, e.g., supply chain network design, deployment of health care facilities, and design of storage systems for renewable power. In this monograph, we perform a review of some important concepts in this emerging stream of literature. Motivated by supply chain design applications, we first discuss the basic modeling concepts, including both mathematical programming-based and analytical approaches for modeling. While simulation-optimization approaches can be used for analyzing location problems, they are not covered in the scope of this monograph. We also review techniques adopted in the literature to analyze and solve these classes of location models. This is aimed to serve as a reference for readers (especially students) who like to develop their own models but are less familiar with this line of research. Furthermore, we review a number of applications of this line of research, covering both applications in supply chain contexts and other emerging domains, such as sustainable transportation, energy and health care.

# 1

# Introduction

Facility location is one of the most crucial strategic planning decisions for governments, firms and non-profit organizations alike. A popular saying in real estate is that the three most important attributes of a property are its *location, location, location.* In marketing, *place* (i.e., location) is considered as one of the four building blocks (“four P’s”) of a marketing strategy, along with price, product and promotion. For firms selling tangible products and services alike, strategic location planning is often the basis of firms’ competitive advantages. For retail stores and service facilities, good location planning allows customers to access the firm’s offering with low access or inconvenience costs (e.g., in the form of travel cost or time), and thereby enhances customers’ willingness to pay and the firm’s revenue. For back-end support facilities such as distribution centers and warehouses, a carefully located network of facilities serves as the backbone for efficient logistics operations. In the public sector, choice of locations of public service facilities (or equipment, as mobile facilities), such as hospitals, fire stations and ambulances, plays a critical role in determining the level of service provided to the public, such as response times to emergency calls.

While facility location can deliver positive strategic value to firms (or organizations), it also poses significant planning risk in a large variety of applications due to the often hefty resource commitment involved. For example, in selecting a distribution center location, the firm needs to acquire (purchase or engage in long-term lease) a piece of land, construct (or acquire) a building, and equip the building with necessary labor and equipment. As a result of this heavy commitment, facilities are very costly to be relocated or closed after they start operating. This high cost of recourse calls for foresight in planning, particularly in forecasting the future operating environment (e.g., demand and costs) and in understanding the long-term operating characteristics of, as well as possible interactions between, the facilities.

Strategically, various considerations underpin the choice of facility locations. Proximity to markets and/or suppliers, operational efficiency of logistics operations (e.g., to replenish stock at the chosen retail store locations), availability of skilled labor or natural resources, access to free or low-tariff trade zones, presence of favorable tax or regulatory policies, political stability of the region, etc., are examples of important factors to evaluate when planning for a network of facilities. As Daskin (2011) (Chapter 1) suggests, these include quantifiable and non-quantifiable factors, and the focus of developing mathematical models is on the former.

Among those quantifiable factors of consideration, the trade-off between service level and cost pertains in the majority of location planning scenarios. Service level refers to the accessibility of facilities by their users, and is typically determined by factors such as response time, and the costs and inconvenience of access. Note that these factors are decreasing in the distances between users and facilities; that is, service level is typically improved as a denser facility network is deployed. Therefore, to maximize user accessibility, a ubiquitous location strategy, where users never need to travel long distances to access the nearest facilities, could be desirable. Examples of this strategy include those adopted by Seven-Eleven in certain densely populated (especially Asian) large cities, or Starbucks in major North American cities. While such strategies make facilities extremely accessible, the obvious downside is the higher operational costs due to the lack of economies of scale in operating each facility.

On the other hand, operating costs of the facility network depend on a multitude of factors. In the majority of planning scenarios, the overall costs consist of fixed and variable components. It is particularly important to note that many facility types (e.g., factories, hospitals, transportation terminals) employ expensive equipment and thus the fixed component of costs is typically sizable. Therefore, the dominant factor in the strategic consideration of operating costs is often economies of scale. That is, operating costs can often be reduced by deploying a network with fewer (i.e., sparser) facilities each handling a larger volume of demand. An example is the “four corners” strategy commonly adopted by North American retailers that operate small numbers of distribution centers, typically near the major East and West Coast ports, to serve demand from the entire continent.

Naturally, the goals of improving service level (which calls for locating more facilities) and reducing operating costs (locating fewer facilities) are in conflict. The early literature focuses on developing optimization models that attempt to balance these goals in different planning contexts. We shall review some of the classical models in the next section.

## Brief Review of Classical Location Theory

In this section, we briefly review some of the most common location models used in practice. Typically, the planner is faced with the problem of locating a number of facilities to serve a discrete set of spatially dispersed customers. Many classical facility location models are formulated to deliberately characterize the trade-off between access distance and costs. Access distance refers to a measure of the distance between customers and the facilities that they patronize, and reflects the design quality of service. Two popular measures of access distance employed in the literature are demand-weighted distance and coverage distance. We shall review these concepts and some of the associated optimization models below.

## Demand-Weighted Distance Models

Demand-weighted distance is a popular metric for access distance considered in facility location models. Particularly, it considers the weighted average of distances between individual customer locations and their respective assigned (or patronized) facilities. Often, the weights are selected to be proportional to the volumes of demand (e.g., number of potential consumers, forecasted sales volumes, etc.) at the customer sites. The consideration of such weights allows the decision maker to prioritize service provision to customers in the sense that facilities tend to be located closer to more important customers with larger weights. In the case where the costs of serving a customer location are bilinear in the location's demand volume and access distance, demand-weighted distance reflects the system-wide operations costs of serving all customers with the assigned facilities. One example is a supply chain setting in which facilities are distribution centers (DCs) and customers are retail stores. Demand-weighted distance, in this case, provides a proxy for the total transportation costs under direct shipments, such that the costs of shipping to one retailer location are approximately given by the shipment volume (demand) times the shipment distance. Below, we briefly review the classical location models that incorporate demand-weighted distance objective.

The $P$-median problem, originally formulated by Hakimi (1964, 1965) is concerned with minimizing the demand-weighted distance of serving a set of customers by locating a given number ($P$) of facilities. Note that in graph theory terminology, the *absolute median* of a network is a point from which the sum of weighted distances to all nodes of the network is the smallest. Thus, the problem of finding the set of $P$ locations that minimize the total demand-weighted distance is referred to as the $P$-median problem. To formulate the problem, we define the following notation:

*Sets*

$I$ = set of customers;

$J$ = set of candidate facility locations.

*Demand and Cost Parameters*
$\mu_i$ = demand volume at customer location $i$, $i \in I$;
$d_{ij}$ = distance between locations $i$ and $j$, $i \in I, j \in J$
$P$ = number (budget) of facilities to be located.

*Decision Variables*
$X_j = 1$ if facility is opened at location $j \in J$, 0 otherwise;
$Y_{ij} = 1$ if facility at $j \in J$ is assigned to serve customer location $i \in I$.

The problem is to select, out of the candidate set $J$, some $P$ facilities, and assign them to serve customers in set $I$. These decisions are indicated by the $X_j$ and $Y_{ij}$ binary decision variables, respectively. In the $P$-Median problem formulation provided below, the objective is to minimize the total distance between customers and their assigned facilities, weighted by demand (1.1). The constraints stipulate that each customer location must be assigned to one facility (1.2), that such assignment can only be made if said facility is opened (1.3), and that the number of facilities opened equals $P$ (1.4).

$$
\begin{aligned}
[P\text{-Median}] \quad \min \quad & \sum_{i \in I} \sum_{j \in J} \mu_i d_{ij} Y_{ij} && (1.1)\\
\text{s.t.} \quad & \sum_{j \in J} Y_{ij} = 1 \text{ for } i \in I && (1.2)\\
& Y_{ij} - X_j \leq 0 \text{ for } i \in I, j \in J && (1.3)\\
& \sum_{j \in J} X_j = P && (1.4)\\
& X_j \in \{0,1\} \text{ for } j \in J\\
& Y_{ij} \in \{0,1\} \text{ for } i \in I, j \in J.
\end{aligned}
$$

For various properties and solution heuristics of the $P$-median problem, one may refer to, e.g., the recent review by Daskin and Maass (2015). A closely-related model is the uncapacitated fixed charge facility location model, which is often also referred to as the uncapacitated facility location (UFL) model. In this model, the hard budget constraint (1.4) is relaxed; instead, opening a facility at site $j \in J$ incurs a fixed cost of $f_j$. By considering an objective function that combines the fixed costs of opening facilities and the distance-based costs of serving customers, the

UFL model may provide a more flexible characterization of the trade-off between the budget of locating facilities and access distance. Let $\rho$ be the unit cost of serving one unit of customer demand per unit distance between the customer and the assigned facility (e.g., unit shipping cost). Then, the uncapacitated fixed charge location model can be formulated as follows:

$$\begin{aligned} [\text{UFL}]: \quad & \min \quad && \sum_{j \in J} f_j X_j + \rho \sum_{i \in I} \sum_{j \in J} \mu_i d_{ij} Y_{ij} && (1.5) \\ & \text{s.t.} && (1.2), (1.3). \end{aligned}$$

It is also noted that both the $P$-median and UFL models do not consider capacity of facilities (e.g., available land area for warehouses). Let $C_j$ be the maximum demand volume that can be handled by a facility at $j \in J$. The capacitated fixed charge facility location model (CFL) is formulated by adding the following capacity constraint, which limits the volume of customer demand that can be assigned to a facility, to the UFL model:

$$\sum_{i \in I} \mu_i Y_{ij} \leq C_j \text{ for } j \in J. \tag{1.6}$$

In generalizing the UFL to the CFL model, one consideration of note is the modeling of single versus multiple sourcing. In the UFL model, one may note that the constraints that $Y_{ij}$ must take on binary values can be relaxed without loss. This is because, given binary values of $X_j$, the remaining problem in the $Y_{ij}$ variables is a bipartite assignment problem, which is a special case of the minimum cost flow problem. Thus, the basic feasible solutions (in $\mathbf{Y}$) are naturally integer-valued (see, for example, Section 11.4 of Ahuja *et al.* (1993) for more detailed discussions). This suggests that, under the UFL setting, it is always optimal to serve all demand from a customer site to the same facility, i.e., use single sourcing. In fact, it can be observed that it is always optimal to assign all demand at a customer location to the nearest open facility. In the CFL model, however, due to the additional capacity constraint (1.6), such closest assignment may not necessarily be feasible. Then, the distinction between single and multiple sourcing becomes relevant. If the application allows demand volume at the same customer site to be split in proportions (given by $Y_{ij}$) among multiple facilities, one may relax the binary constraints on $Y_{ij}$ to simply $0 \leq Y_{ij} \leq 1$, which potentially improves the objective value.

### Coverage Distance

The demand-weighted distance objective provides an average-case view (over the set of customers) of the facility network, by considering the aggregate service measure (measured by access distance) provided to all customers, weighted by demand sizes. This may not be the most appropriate objective in applications where the worst-case service provision to customers is of primary concern. For example, for emergency medical services, the planning objective is often to maximize the volume or proportion of potential demand that can be served within a prescribed time guarantee, rather than the average response time to requests. Similar considerations arise in retail settings, where stores can attract customers located within certain distances. In these applications, the primary concern in planning is whether or not a facility is available within a certain critical distance, which is referred to as the coverage distance, to each customer.

To reflect whether a customer is located within the coverage distance, denoted by $d_C$, of a facility, we define the binary parameter $a_{ij} = \mathbf{1}(d_{ij} \leq d_C)$, where $\mathbf{1}(\cdot)$ denotes the indicator function. Then, we can formulate the set covering location model (Toregas *et al.*, 1971), which aims to locate the minimum number of facilities to cover all customers within the coverage distance.

$$
\begin{aligned}
\text{[Set Covering Location]}: \quad & \min && \sum_{j \in J} f_j X_j && (1.7)\\
& \text{s.t.} && \sum_{j \in J} a_{ij} X_j \geq 1 \text{ for } i \in I && (1.8)\\
& && X_j \in \{0, 1\} \text{ for } j \in J. &&
\end{aligned}
$$

The objective (1.7) is to minimize the number (or more generally, opening costs) of facilities required to satisfy constraints (1.8) that require at least one facility to be opened within the coverage radius from each customer location. The set covering location problem has important applications in the public sector. For example, the location of facilities such as hospitals, emergency medical services, police and fire stations, and schools, all should incorporate the access radius as a primary criterion in planning.

More generally, the set covering problem is one of selecting an optimal (minimum-cost) set of subsets of a collection of elements under the constraint that all elements have to be covered in at least one selected subset. In the facility location context, the elements refer to customer locations, and each feasible subset of elements is defined as the group of customers within the coverage distance of each candidate facility location. Thus, selecting among these subsets of customers is equivalent to selecting among candidate locations. Furthermore, the constraint that all elements are included in selected subsets is interpreted as requiring all customers to be covered within the prescribed coverage distance from some selected facilities.

The general formulation for set covering (Roth, 1969) is provided as follows. Let $I$ be the set of elements to be covered, and $N \subseteq 2^I$ be a collection of feasible subsets of $I$. Then, for each member $R \in N$, we define the binary decision variable $Z_R$ to indicate whether the set $R$ is selected or not, with the cost associated given by $c_R$. Then, the general set covering problem can be formulated as:

$$\begin{aligned}
\text{[General Set Covering]} : \quad & \sum_{R \in N} c_R Z_R && (1.9)\\
\text{s.t.} \quad & \sum_{R \in N : i \in R} Z_R \geq 1 \text{ for } i \in I && (1.10)\\
& Z_R \in \{0,1\} \text{ for } R \in N.
\end{aligned}$$

Interestingly, the general set covering problem arises in the solution procedure of some class of integrated location models with weighted-distance objectives. We shall revisit this in Section 3.

One limitation of the set covering location problem is its strict requirement that all customers must be covered, which was appropriate in the original context studied by Toregas *et al.* (1971) of locating emergency service facilities. While this requirement is often necessary for public sector services, we note that it is often the case that the marginal demand coverage for increasing the number of facilities is decreasing. Thus, in settings involving planners in the private sector, it is often beneficial to leave out certain customers that are too costly to cover. One may then consider the maximum covering problem that maximizes demand coverage subject to a given budget to locate facilities, formulated as follows:

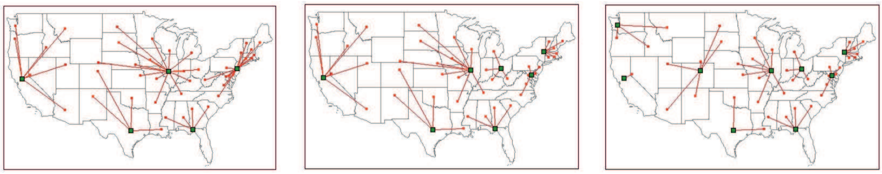
$$\begin{aligned}
\text{[Max Covering Location]}: \quad \max \quad & \sum_{i \in I} \mu_i U_i && (1.11)\\
\text{s.t.} \quad & \sum_{j \in J} a_{ij} X_j \geq U_i \text{ for } i \in I && (1.12)\\
& \sum_{j \in J} X_j \leq P && (1.13)\\
& U_i, X_j \in \{0, 1\} \text{ for } i \in I, j \in J.
\end{aligned}$$

In the above, the objective (1.11) is to maximize the volume of demand being covered by the network of facilities, where binary decision variable $U_i$ indicates whether customer location $i$ is covered. Constraints (1.12) are similar to (1.8) in the set covering problem, but allow the flexibility of not covering certain customer locations, in which case they do not contribute to the objective ($U_i = 0$). Constraint (1.13) limits the number of facilities to the budgeted number ($P$).

## Motivation for Integrated Modeling

The location models discussed so far focus on the fundamental trade-off between facility location costs and access distance. Despite the strategic importance of this trade-off, we may observe in a variety of applications that this alone is inadequate to capture other important strategic considerations in location design. Here, we provide an illustration based on a supply chain design setting.

Consider the problem of deploying DCs to serve a geographical market (e.g., the contiguous US). For illustration, we use the 49-node data set provided by Daskin (2011). The 49 nodes, which serve as both customer locations and candidate facility locations are the state capitals of the 48 contiguous states and Washington DC. The demand rates at each of these customer nodes are assumed to be proportional to the state populations and the shipping costs are proportional to great circle distances[1] between the cities. Following the classical modeling approach, one might determine the locations based on the UFL model, by considering location costs $f_j$ as the (annualized) construction and operating costs of the DCs and $\rho d_{ij}$ as the shipping cost per unit demand between two locations $i$ and $j$. To illustrate the trade-off between location and transportation costs, we vary the weight $\rho = 1, 1.5, 2$ on the unit transportation cost and compare the optimal DC locations, as mapped in Figure 1.1. Intuitively, a higher transportation cost weight leads to locating more DCs, as higher unit transportation costs favors reducing shipping distances from DCs to customers by increasing the density of DCs. In general, the relative magnitudes of the location cost and transportation cost weights determine the degree of *consolidation* of the supply chain network.



(a) $\rho = 1$ (b) $\rho = 1.5$ (c) $\rho = 2$

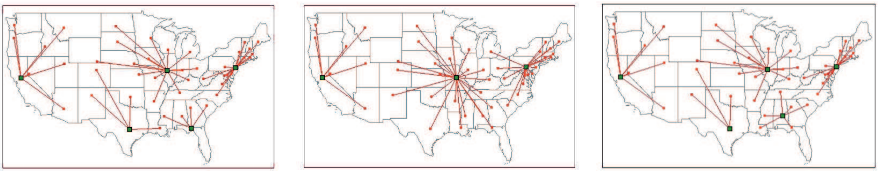
**Figure 1.1:** UFL Solutions Under Different Transportation Costs

However, one may notice that the aforementioned consideration does not fully capture the consolidation-deconsolidation trade-off in strategic distribution network design. In supply chain management, it is well known that facility costs, transportation costs and inventory costs are the three major cost components driving network design decisions (e.g., Chopra and Meindl, 2007). The conventional models focus on the former two, but do not account for inventory costs. To illustrate why this can be a problem, we extend the example by comparing the UFL setting with two other alternative settings that consider inventory costs.

---

[1] The great circle distance is the shortest distance between two points on a sphere. It is often used as a proxy for the straight-line distance between two cities, adjusted for the Earth's surface curvature.

Figure 1.2 (a) shows the solution to the UFL problem for the dataset (with $\rho = 1$), which consists of five opened facilities in Sacramento (CA), Austin (TX), Tallahassee (FL), Springfield (IL) and Trenton (NJ). We refer to the UFL problem as Setting 1 and the corresponding optimal solution (set of chosen facilities) as Solution 1. To account for inventory costs, consider a setting (Setting 2) in which demand is random (with mean and standard deviation proportional to state population). Each facility, once located, needs to carry enough safety stock to ensure a 95% Type-1 service level. Under this alternative model, we may solve a stochastic optimization model to obtain the optimal solution (Solution 2) illustrated in Figure 1.2 (b). One can observe that there are now only three DCs instead of five.



(a) UFL Solution (b) Service Level (c) Transshipments

**Figure 1.2:** Maps of Location Plans under Different Model Settings

One may naturally wonder why locating three DCs rather than five (at different locations) would be optimal as one considers safety stock holding costs. One major reason is the effect of risk pooling (Eppen, 1979). In particular, safety stock can be reduced by pooling larger volumes of demand at smaller number of DCs. This “statistical” economies of scale effect tilts the optimal balance in the consolidation-deconsolidation trade-off and causes the optimal number of DCs to be reduced. A more detailed discussion of such effects will be provided in later chapters. To make things even more interesting, we consider another alternative setting (Setting 3) in which facilities may transship inventory among themselves to cope with random demand. Furthermore, instead of satisfying a Type-1 service level, the safety stock level is chosen to minimize a newsvendor-type cost function including holding, shortage, and transshipment costs. The resulting optimal solution (Solution 3) is provided in Figure 1.2 (c). Interestingly, not only is it optimal to locate five

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rather than three DCs, but the locations are also slightly different from Solution 1; in particular, Montegomery (AL) is selected instead of Tallahassee (FL). This is, in part, due to the consideration of transshipment operations. First, with the possibility of sharing inventory through transshipments, it is possible to share inventory and achieve pooling benefits without the need to deliberately consolidate the network of DCs (i.e., reducing to four DCs in the case of no transshipments). Second, the specific locations of the five DCs is the outcome of the trade-off between minimizing transportation costs to customers and transshipment costs. The former encourages DCs to be located closer to centers of customer clusters, and the latter encourages DCs to be placed closer to each other. With the additional consideration of the transshipment effect, the choice of Montegomery (AL) allows the set of DCs to be, on average, more centrally located within the country.

**Table 1.1:** Percentage Performance Gaps of the Three Solutions Under the Three Settings

| | Solution 1 | Solution 2 | Solution 3 |
|---|---|---|---|
| Setting 1 | 0.00% | 2.15% | 0.23% |
| Setting 2 | 6.29% | 0.00% | 6.19% |
| Setting 3 | 1.95% | 5.90% | 0.00% |

Note that the optimal strategy in one setting is suboptimal in the others. In Table 1.1, we compare the performance of each of the three solutions under each of the three settings. In particular, we report the percentage cost increase of each solution over the optimal one in the same setting. We observe that both Solutions 1 (6.3% worse than optimal) and 3 (6.2% worse than optimal), which suggest opening five DCs, perform substantially worse in Setting 2 than the optimal solution (Solution 2). This suggests that failure to account for the risk-pooling effect leads to significant cost increases. On the other hand, Solution 2 also performs relatively poorly under Setting 3, suggesting that failure to account for transshipment opportunities at the network design stage also leads to cost inefficiencies. Finally, although Solution 1 differs from Solution 3 by the location of just one DC, it performs about 2% worse under

Setting 3. This further highlights that importance of selecting the right set (on top of the right number) of facilities for the problem setting on hand.

From this simple illustrative example, we can see that conventional models that consider generic, distance-only objectives (e.g., the UFL model) may fail to capture important design characteristics arising from specific operations of certain facility types, leading to significantly sub-optimal network designs. This potential shortcoming can be overcome by enhancing the models with an integrated view of both the conceptual cost-distance trade-off and the operating characteristics of the specific facility types. This monograph is dedicated to reviewing the recent developments of this line of research.

## Aims and Scope

While we have briefly introduced the classical facility location models in Section 1, we do not attempt to provide a comprehensive review of this extensive literature. Our focus will be on integrated models that incorporate operational features of facilities beyond distance-focused considerations. For more comprehensive reviews and discussions of the properties and solution strategies for classical models, as well as various extensions, applications and modeling discussions, one may refer to the excellent texts by Daskin (2011), Drezner (1995), Hamacher and Drezner (2002), and Laporte *et al.* (2015). Likewise, while many of the applications we shall discuss make use of important results in research streams such as inventory theory to model operational features of facilities, we also do not intend to provide a full review of these areas beyond what is required to develop the integrated facility location models. Interested readers may refer to, for example, Zipkin (2000), for more complete discussion and references.

The study of integrated facility location modeling has a long history. In the 1980's, works by Daskin (1983), Eaton *et al.* (1985), and ReVelle and Hogan (1989) consider the operational characteristics of mobile facilities such as ambulances and the optimal deployment strategies taking into account congestion probabilities. However, it was until the 2000's when this research area sustained very rapid growth. Part of

Full text is available at http://dx.doi.org/10.1561/0200000037

the reason was the significant computational challenges associated with solving the integrated models, which had been difficult to overcome before the recent advancements in computational power of computers as well as optimization (particularly, stochastic and nonlinear integer programming) theory. With the rapid growth, a myriad of modeling approaches, solution methodologies and application areas have been proposed by researchers and practitioners. The aim of this monograph is to provide a timely review of some of these important developments. With the exploding growth and huge volume of related research, our review is inevitably restricted in scope and cannot be comprehensive. As our aim is to review major modeling approaches, solution methodology and some promising current and future research directions, some application areas are inevitably omitted. For other recent reviews, we refer interested readers to Shen (2007) and Mak and Shen (2011). It is also notable that simulation-optimization techniques, designed for ranking and selection problems where performances of alternatives can be evaluated via simulation, are also a promising approach to the class of problems that we consider, since the operational performance of facilities can be simulated in detail. Our focus will be mainly on mathematical programming and analytical modeling approaches, and refer interested readers to Fu (2002), Hong and Nelson (2009), and Luo *et al.* (2015) (and the references therein) for this alternative methodology.

In this monograph, we provide discussion on four aspects of the research stream. In Chapter 2, we discuss several popular modeling approaches employed by researchers to model integrated location problems, such as nonlinear integer programming, stochastic programming and continuous approximation. In Chapter 3, we provide a brief account of some promising solution methodologies, including decomposition methods and conic optimization methods. Then, in Chapters 4 and 5, we draw from the broad range of applications of the integrated modeling framework in the classical supply chain design context and several other emerging application domains, respectively. Finally, we conclude the volume and discuss some promising future research directions in Chapter 6.

## Notation

Throughout the monograph, we use boldface letters to represent matrices or vectors of variables denoted by the same letters. For example, $\mathbf{Y}$ is the matrix with components being the $Y_{ij}$'s. Furthermore, $\mathbf{x}'$ denotes the transpose of column vector $\mathbf{x}$, and $\mathbf{x}'\mathbf{y}$ denotes the inner product of column vectors $\mathbf{x}$ and $\mathbf{y}$.

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